The examination of these very important and interesting particulars has, I observe, drawn me into a prolixity, which I fear may prove tedious to your Lordship: but should I have removed all doubts, and brought convincing proofs of the absurdity of fearing a second infection; should I have shewn inoculation to be a necessary practice, and that the contagious distemper may be communicated more ways than one; I hope your Lordship will excuse the length of this letter. I shall only add my earnest wishes, that the legislature may, by effectual means, prevent the importation of distempered cattle and hides into these kingdoms; the only means of naturalizing and perpetuating a dreadful distemper, now, thank God! much decreased among us.

I am, with the greatest respect,

My Lord,

Your Lordship's

Most humble and most obedient Servant,

Huntingdon,
26 Nov. 1757.

Daniel Peter Layard.

LXX. *Trigonometry abridged. By the Rev.* Patrick Murdoch, *A. M. F.R.S.*

Read Feb. 2, 1758.

THE cases in trigonometry, that can properly be called different from one another are no more than *four*; which may be resolved by *three* general rules or theorems, expressed

5 in

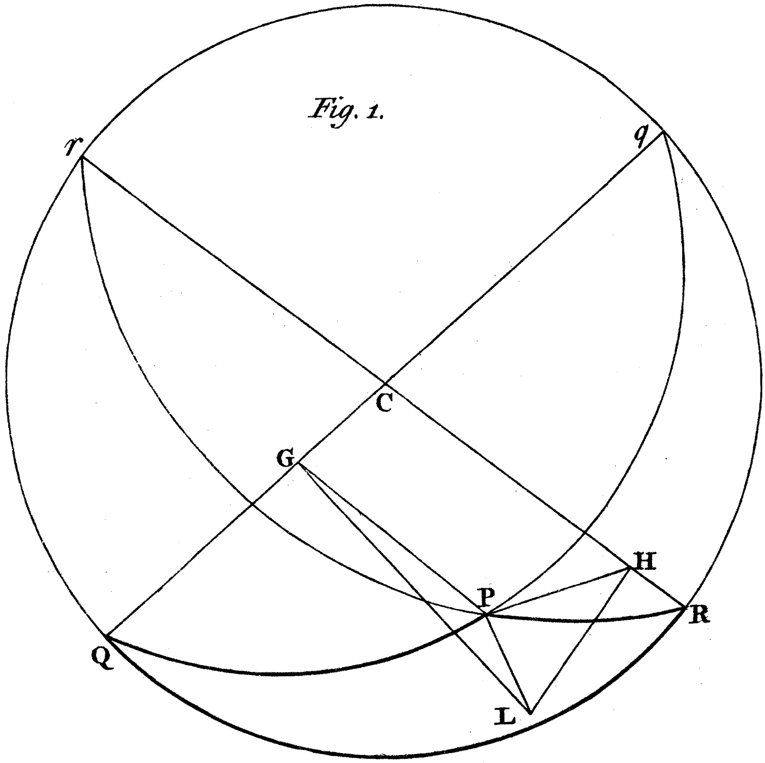

Fig. 1.

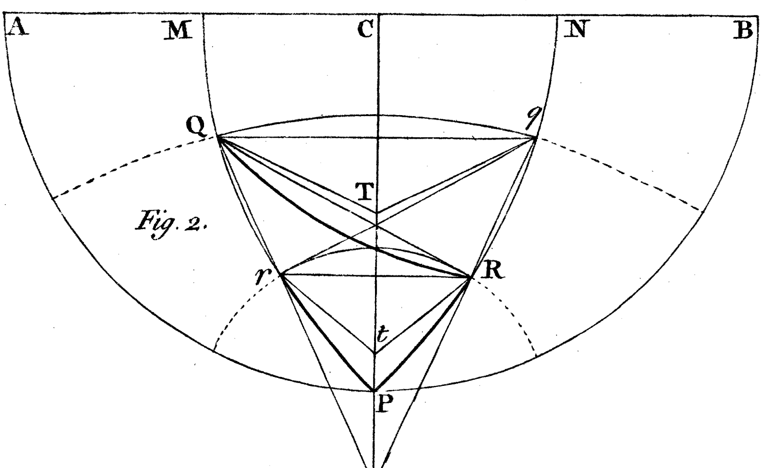

Fig. 2.

J. Mynde sc.

in the ſines of arcs only; uſing the ſupplemental triangle as there is occaſion.

CASE I.

When of three given parts two ſtand oppoſite to each other, and the third ſtands oppoſite to the part required.

THEOREM I.

The ſines of the ſides are proportional to the ſines of angles oppoſite to them.

DEMONSTRATION.

Let QR (TAB. XX. *Fig.* 1.) be the baſe of a ſpherical triangle; its ſides PQ, PR, whoſe planes cut that of the baſe in the diameters QC*q*, RC*r*. And if, from the angle P, the line PL is perpendicular to the plane of the baſe, meeting it in L, all planes drawn through PL will be perpendicular to the ſame, by 18. *el.* 11. Let two ſuch planes be penpendicular likewiſe to the ſemicircles of the ſides, cutting them in the ſtraight lines PG, PH; and the plane of the baſe in the lines LG, LH.

Then the plane of the triangle PGL being perpendicular to the two planes, whoſe interſection is QGC*q*, the angles PGQ LGQ will be right angles, by 19. *el.* 11. PG likewiſe ſubtends a right angle PLG, and the angle PGL meaſures the inclination of the ſemicircle QP*q* to the plane of the baſe (*def.* 6. *el.* 11.) that is (by 16 *el.* 3. and 10 *el.* 11.) it is equal to the ſpherical angle PQR: whence PG is to PL as the radius to the ſine of PQR. The ſame way PL is to PH as the ſine of PRQ is to the

the radius: and therefore, *ex æquo*, PG the fine of the fide PQ is to PH the fine of PR, as the fine of PRQ is to the fine of PQR.

CASES II. *and* III.

When the three parts are of the fame name:

And,

When two given parts include between them a given part of a different name, the part required ftanding oppofite to this middle part.

THEOREM II.

Let S *and* s *be the fines of two fides of a fpherical triangle,* d *the fine of half the difference of the fame fides,* a *the fine of half the included angle,* b *the fine of half the bafe; and writing unity for the radius, we have* $Ssa^2 + d^2 - b^2 = 0$; *in which* a *or* b *may be made the unknown quantity, as the cafe requires.*

DEMONSTRATION.

Let PQR (*Fig.* 2.) be a fpherical triangle, whofe fides are PQ, PR, the angle included QPR, the bafe QR, PC the femiaxis of the fphere, in which the planes of the fides interfect.

To the pole P, draw the great circle AB, cutting the fides (produced, if needful) in M and N; and thro' Q and R, the leffer circles Q*q*, *r*R, cutting off the arcs Q*r*, *q*R equal to the difference of the fides; join MN, Q*q*, *r*R, QR, *qr*.

Then the planes of the circles defcribed being parallel (*Theod. fphæric.* 2. 2.), and the axis PC perpendicular to them (10. 1. *of the fame*), their interfec-tions

tions with the planes of the ſides, as QT, and Rt, will make right angles with PC; that is, QT and Rt are the ſines (S, s.) of the ſides PQ PR, and MC NC are whole ſines. Now the iſoſceles triangles MCN, QTq, rtR, being manifeſtly ſimilar; as alſo MN, the ſubtenſe of the arc which meaſures the angle QPR, being equal to ($2a$) twice the ſine of half that angle; we ſhall have MN : MC :: Qq : QT :: rR : Rt; or, in the notation of the theorem, $Qq = 2Sa$, $rR = 2sa$. And further, the chords Qr qR being equal, and equally diſtant from the center of the ſphere, as alſo equally inclined to the axis PC, will, if produced, meet the axis produced, in one point Z. Whence the points Q, q, R, r, are in one plane (2. *el.* 11.), and in the circumference in which that plane cuts the ſurface of the ſphere: the quadrilateral QqRr is alſo a ſegment of the iſoſceles triangle ZQq, cut off by a line parallel to its baſe, making the diagonals QR, qr, equal. And therefore, by a known property of the circle, $Qq \times rR + \overline{qR}^2 = \overline{QR}^2$; which, ſubſtituting for Qq and Rr the values found above, $2d$ for Qr, $2b$ for QR, and taking the fourth part of the whole, becomes $Ssa^2 + d^2 = b^2$, the propoſition that was to be demonſtrated.

Note 1. If this, or the preceding, is applied to a plane triangle, the ſines of the ſides become the ſides themſelves; the triangle being conceived to lie in the ſurface of a ſphere greater than any that can be aſſigned.

Note 2. If the two ſides are equal, d vaniſhing, the operation is ſhorter: as it likewiſe is when one or both ſides are quadrants.

Note

Note 3. By comparing this proposition with that of the Lord Neper ‡, which makes the 39th of Keill's Trigonometry, it appears, that if AC, AM, are two arcs, then sin. $\frac{AC + AM}{2}$ × sin. $\frac{AC - AM}{2} = (\overline{b+d} \times \overline{b-d} =)$ $\overline{\text{sin. } \frac{1}{2} AC + \text{sin. } \frac{1}{2} AM} \times \overline{\text{sin. } \frac{1}{2} AC - \text{sin. } \frac{1}{2} AM}$. And in the solution of Case II. the first of these products will be the most readily computed.

CASE IV.

When the part required stands opposite to a part, which is likewise unknown : Having from the *data* of Case I. found a fourth part, let the sines of the given sides be S, s; those of the given angles Σ, σ; and the sines of half the unknown parts a and b; and we shall have, as before, $Ssa^2 + d^2 - b^2 = 0$; and if the equation of the supplements be $\Sigma\sigma\alpha^2 + \delta^2 - \beta^2 = 0$; then, because $\alpha^2 = 1 - b^2 = 1 - \overline{Ssa^2 + d^2}$, and $\beta^2 = 1 - a^2$, substituting these values in the second equation, we get

THEOREM III.

$\frac{1 - \Sigma\sigma \times \overline{1 - d^2} - \delta^2}{1 - Ss\Sigma\sigma} = a^2$; in words thus:

Multiply the product of the sines of the two known angles by the square of the cosine of half the difference of the sides : add the square of the sine of half the difference of the angles ; and divide the complement of this

‡ See Logarith. Canon. descr. *Edinb.* 1614. p. 48.

sum

ſum to unity, by the like complement of the product of the four ſines of the ſides and angles; and the ſquare root of the quotient ſhall be the ſine of half the unknown angle.

If we work by logarithms, the operation will not be very troubleſome; but the rule needs not be uſed, unleſs when a table of the trigonometrical analogies is wanting. To ſupply which, the foregoing theorems will be found ſufficient, and of ready uſe; being either committed to memory, or noted down on the blank leaf of the trigonometrical tables.

Note, The ſchemes may be better, raiſed in card-paper, or with bent wires and threads.

LXXI. *An Account of Two extraordinary Caſes of Gall-Stones.* By James Johnſtone, *M. D. of* Kidderminſter. *Communicated by the Rev.* Charles Lyttelton, *L. L. D. Dean of* Exeter.

To the Rev. Dr. Lyttelton, *Dean of* Exeter.

Rev. Sir,

Read Feb. 9, 1758.

ACcording to promiſe I ſend you a ſhort account of the two extraordinary caſes we talked of, the laſt time I had the pleaſure of ſeeing you at Kidderminſter.

The truth of the firſt narrated caſe you are already a ſufficient judge of; and if it is at all neceſſary to aſcertain